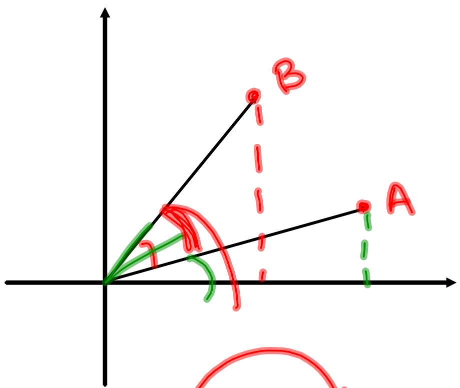
Inner product and cosine



A(4, 1) and B(2, 3)

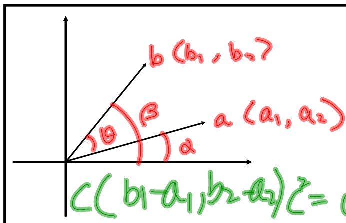
What is the measure of angle AOB?

$$\arctan(3/2) - \arctan(1/4) = .738$$

$$(42.3^\circ)$$

$$A^T B = \|A\|\|B\|\cos\theta$$

$$\frac{11}{\sqrt{17}\sqrt{13}} = \cos\theta$$



$$c(b_1-a_1, b_2-a_2) \quad c^2 = a^2+b^2-2ab\cos C$$

$$a^T b = \|a\|\|b\|\cos\theta$$

$$\cos\theta = \cos(\beta-\alpha)$$

$$\left(\sqrt{(b_1-a_1)^2+(b_2-a_2)^2}\right)^2 = \left(\sqrt{b_1^2+b_2^2}\right)^2 + \left(\sqrt{a_1^2+a_2^2}\right)^2 - 2\sqrt{(a_1^2+a_2^2)(b_1^2+b_2^2)}\cos\theta$$

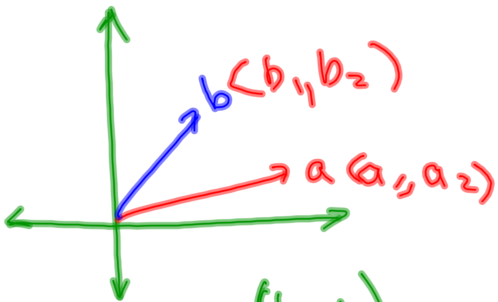
$$(b_1-a_1)^2+(b_2-a_2)^2 = b_1^2+b_2^2+a_1^2+a_2^2 - 2\sqrt{(a_1^2+a_2^2)(b_1^2+b_2^2)}\cos\theta$$

$$\cancel{-2}a_1b_1 \cancel{-2}a_2b_2 = \cancel{-2}\sqrt{(a_1^2+a_2^2)(b_1^2+b_2^2)}\cos\theta$$

$$a^T b = \sqrt{a_1^2+a_2^2}\,\sqrt{b_1^2+b_2^2}\,\cos\theta$$

$$a^T b = \|A\|\cdot\|b\|\cos\theta$$

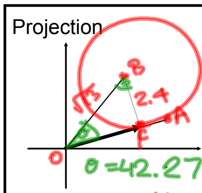
$$a^T b = \|a\| \|b\| \cos\theta$$



$$\cos\theta = \cos(b - a)$$
$$= \cos a \cos b + \sin a \sin b$$

$$\|a\|\|b\|\cos\theta = \|a\|\|b\|\cos\theta$$
$$ab\cos\theta = ab(\cos a \cos b + \sin a \sin b)$$
$$ab\cos\theta = ab\cos a \cos b + ab \sin a \sin b$$
$$ab\cos\theta = a^T b$$
$$a_1 \cdot b_1 + a_2 b_2$$

Projection



A(4, 1) and B(2, 3)

What is the coordinate of C, if angle OCB is 90?

$C = (x_1, y_1)$

$\theta = 42.27$

$b = 180 - (90 + 42.27)$
$= 47.726$

$$\frac{\|C\|}{\|B\|} = \sin\theta, \quad \|c\| = 2.4$$

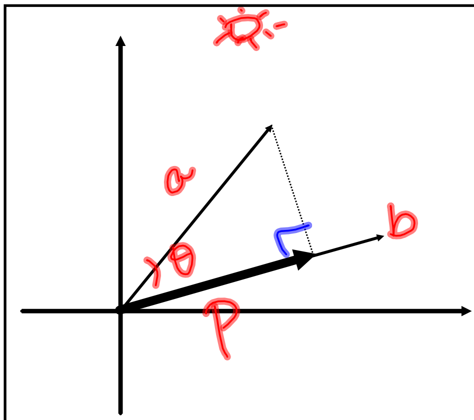
$$\|c\| = \sqrt{(2-x_1)^2 + (3-y_1)^2} = \sin\theta\sqrt{13}$$
$$(2-x_1)^2 + (3-y_1)^2 = (2.4)^2$$

$$(2.4)^2 + d^2 = 13$$
$$d^2 = 7.24$$
$$d = \sqrt{7.24} \approx 2.69$$

$17a^2 = 7.24$ $\quad a = \sqrt{\frac{7.24}{17}} \approx .65$

$4a = 2.6$

$(2.6, .65)$



$$a^T b = \|a\|\|b\|\cos\theta$$

$$p = \frac{a^T b}{\|b\|^2}\cdot b$$

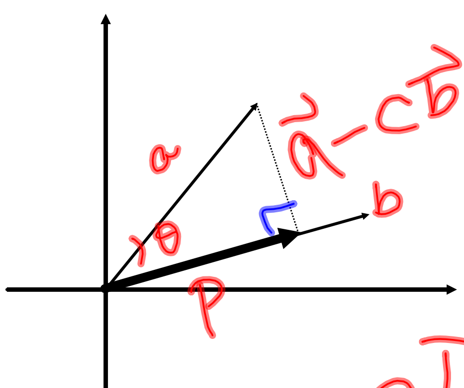
$$\frac{b}{\|b\|} = u$$

$$a\cdot u = \|a\|\cos\theta$$

$$\frac{a\cdot b}{\|b\|} = \|a\|\cos\theta$$

$$\frac{a\cdot b}{\|b\|}\cdot\frac{b}{\|b\|} = \mathrm{Proj}_b a$$

$$\frac{a\cdot b}{\|b\|^2}\cdot b = \mathrm{Proj}_b a$$



$$p = c\vec{b}$$

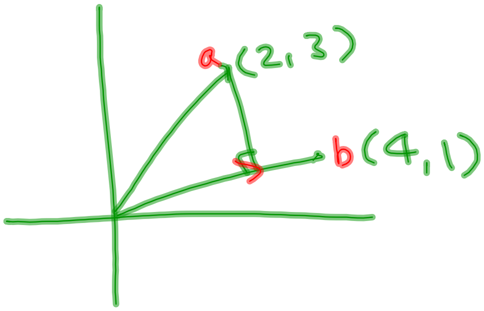
$$(\vec{a} - c\vec{b})\cdot\vec{b} = 0$$

$$a\cdot b - c\,b\cdot b = 0$$

$$a^T b = c\,b^T b$$

$$\frac{\vec{a}\cdot\vec{b}}{\|b\|^2} = c$$

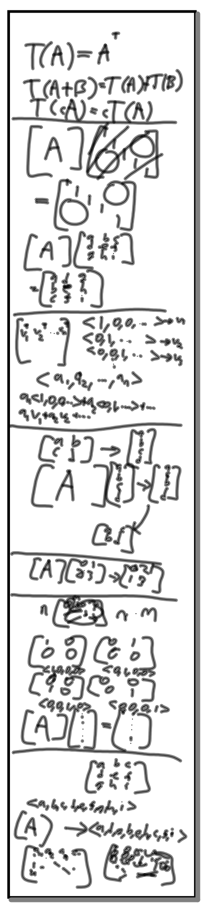
$$\frac{a^T b}{\|b\|^2}$$



$$\frac{a^T b}{\|b\|^2}\cdot b = \frac{11}{17}(4,1)$$

$$= \left(\frac{44}{17}, \frac{11}{17}\right)$$

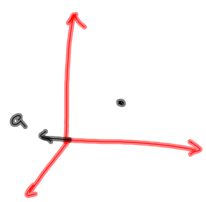
$$=$$



Schwarz inequality $|a^{\mathrm{T}}b| \le \|a\|\|b\|$

What multiple of $a = (1, 1, 1)$ is closest to the point $b = (2, 4, 4)$?

$(1,1,1)x$



$\sqrt{(x_1-y_1)^2+(x_2-y_2)^2\ldots}$

$(2-x)^2+(4-x)^2+(4-x)^2$

$\frac{d}{dx}(3x^2-20x+36)$

$6x-20=0$

$x=\frac{10}{3}$

$\frac{10}{3}$

$$\frac{[2\ 4\ 4]\begin{bmatrix}1\\1\\1\end{bmatrix}}{[1\ 1\ 1]\begin{bmatrix}1\\1\\1\end{bmatrix}}\begin{bmatrix}1\\1\\1\end{bmatrix}=c\vec{a}$$

$$\frac{10}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}=c\vec{a}$$

$$\begin{bmatrix}\frac{10}{3}\\\frac{10}{3}\\\frac{10}{3}\end{bmatrix}=c\vec{a}$$

nd also the point closest to *a* on the line through *b*.

oject the vector *b* onto the line through *a*.

(a) $b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $a = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$